

Guiding the Four Operations

The four math operations - addition, subtraction, multiplication, and division - are the foundations for all the math work that your child will do throughout elementary. In this video, we're going to be exploring many commonly asked questions about operations, including: Where do you start? What sequence should you follow? How do you balance presenting the process of the operations with the work of memorization? How do you keep the interest alive? And how do you support the passage towards mental work, also known as abstraction?

To guide your child through the concepts presented in this video, you'll need to have Montessori albums or the Lesson Sequence Timeline. In the video notes I'm also including several external resources, mini-lessons, and a handy math glossary. I want this video to be something you can refer to again and again, so please don't think you have to absorb and apply everything right away! It's a lot to explore but we can do this together, so let's get started!

WHAT ARE OPERATIONS?

The four operations are basically shortcuts; just faster ways of counting. Think about it, if you have eight apples and you harvest twelve more, you could count one by one from nine through 20, or you could just add 8 plus 12 and get 20. The same applies to the other three operations. And our decimal system is ridiculously easy because it just repeats the numbers one through nine in different categories. So if your child can count to 9, they can do the four operations!

WHY DO WE USE FOUR-DIGIT NUMBERS?

To a second-plane child, four-digit numbers are a lot more interesting than one-digit numbers. They also give a more impactful visual and tactile experience during lessons, and they provide built-in repetition, which is so important for elementary children! If you think about it, a child who's doing one four-digit addition problem is actually doing four little additions...And a child doing ONE checkerboard problem with a four-digit multiplicand and four-digit multiplier is actually working out sixteen multiplications and about a dozen additions!

WHERE DO WE START?

Our launch pad is the work of making and reading numbers with the Golden Beads material. This same material will also give the child their first experience for all four operations with the Golden Beads. But this is not where they're going to truly understand the ins and outs of the operations.

The work with the Golden Beads material is not meant to be done independently; in Primary it's actually called 'Collective Exercises' because it's meant to be used in the company of others. Dr. Montessori designed the Stamp Game to allow children to work on their own, and this is what they'll use once they have a good grasp of place value and have experienced the process of the four operations with the Golden Beads.

We always start the journey of operations in the land of addition, and we stick with addition until the child transitions to working only on paper. This could take days, weeks, or months, and all these timelines are ok.

Once your child is confident doing addition on paper, you can follow their interests towards multiplication or subtraction. Or you can take a break and explore another math concept, such as fractions, measurement, or graphing, for a few weeks. By incorporating your child's new skills into the work they do in other areas of math, you ensure that they retain their new knowledge and apply it in different contexts.

HOW DO WE PRESENT?

As a rule, we need to remember that we're not the teacher; the material is. We're just here to show the child how to use it, but they need to use their brain to understand the concept, and they need to practice repeatedly and independently, until eventually they can let go of the material and achieve *abstraction*, or the ability to work directly on paper. This approach can be hard to follow for many reasons. First of all, you probably weren't taught math this way, and when you feel stressed or uncertain, you tend to teach as you were taught. Secondly, you haven't seen spontaneous abstraction happen so you don't know what to look for. And then there's the annoying fact that learning is not linear and it happens in spurts, which can leave you wondering if your child is making any progress or if you should show them a little shortcut or trick to hurry them along. It's so tempting to think for them, isn't it, just like it was tempting to buckle a shoe or zip up a jacket back when they were three. But we need to respect their right to do the mental work, and we should only offer support when necessary.

The best thing you can do to give an impactful presentation is to practice with the material before you give the lesson. Put yourself in your child's shoes and think about what they would experience. Even better, grab another adult and give them the lesson. The more prepared you are, the more confident you'll be, and you won't be tempted to over-explain.

The best way to present a new concept is to show one or two problems while your child watches, then do one or two problems together, and then let them do one or two problems on their own while you watch. When you're presenting to your child, don't explain in detail what you're doing, just speak enough to direct their attention to your actions. You want your words to become their inner voice, and if you clutter their mind with too many words, they won't retain them. Let the child watch and help, and let them use their reasoning mind to make sense of what they're seeing, but don't quiz them. Then, let them take over so you can focus on observing how well they've understood. Remember that at first we're focusing on the process, so if they make a counting mistake, let it go. You can learn more about guiding from process to accuracy to mastery in the video "The Value of Mistakes".

HOW TO SUSTAIN INTEREST

If you're sure your child understands what they're doing, but they don't want to use the material after a few days of practice, they could be encountering one of three roadblocks:

1. **The work isn't relevant to them:** In this case, bring in word problems based on your child's interests, tell them a story about the history of that operation (you can do a quick online search during your prep time), or use that operation in calculations for geography, history, or money work.
2. **There isn't enough variety:** If they're tired of always using the same material, switch things up by exploring that operation through another relevant material (like a bead frame), or connecting math to geometry. For example, you can use multiplication to explore the concept of area or addition to calculate the perimeter of shapes.
3. **There's not enough challenge:** In this case, throw in an interesting case, like a zero in the minuend in subtraction or in the dividend in division. You can also explore addition through the Fibonacci numbers, or just play at doubling numbers and see how quickly you can get into the thousands!

MEMORIZATION

As your child learns the process of a four-digit operation, you'll also be helping them memorize the facts for that operation. However, please don't use the term "memorization" with your child, because our goal is not to drill these facts into their mind. Instead, we want them to discover how much they're able to retain by doing hands-on learning activities. The strategy we use for the discovery and retention of facts is repetition with variety. You can introduce all the following activities to your child: games with the colored beads, games with the Montessori charts, special cases, math songs, playing card games, dice games, and simple word problems focusing on facts. You'll find the bead games, chart activities, and special cases in your albums or in the Timeline, and I'll link to some card and dice games, songs, and word problems in the video notes.

MENTAL COMPUTATION

To help your child master facts, it is often useful to present strategies for mental computation. Interestingly, some children figure these out on their own, while others need more guidance. Please note that using fingers to count is NOT a bad thing and should NOT be discouraged. But, we also want to give our children tools so they can become more efficient thinkers. Mental computation strategies should be presented as isolated mini-lessons only after they've had plenty of time to explore with the beads, charts, and other memorization activities. You'll find these mini-lessons linked in the lesson notes.

PUTTING IT ALL TOGETHER

I know it sounds like a lot to take in, but this is why we stick to one operation at a time. So, what should a month of math look like? It'll be different for each child! But in general, here's what I suggest: Start by presenting the operation with the Golden Beads, doing the operation without exchanges one day and then introducing the concept of exchanges the next day. When you see that they're able to follow the steps with the Golden Beads, it's time to switch to the Stamp Game.

The focus at first should be on the process, so stick to four-digit numbers and observe to see if they're using the material the way you showed them. Then you can start sprinkling in games on

alternate days, which will support the memorization of facts, and this will add variety right when they start getting bored of practicing the process every day. And then you'll continue moving back and forth between work that focuses on the process and work that focuses on memorization, offering your child choices and ensuring they're maintaining a good balance. Watch to see if they're applying their knowledge of facts to their work with the Stamp Game, Bead Frame, Checkerboard or other process-based materials. And when they start skipping the material and working directly on paper, look for accuracy and understanding.

PASSAGE TO ABSTRACTION

There might be a period of time where they'll want to try working abstractly but their work won't be quite accurate. Instead of correcting them, suggest that they use the material as a calculator to check their work. In other words, let them work the problem out on paper, but then ask them to do the work with the material and compare answers. If their answer on paper doesn't match what they got with the material, they can compare what they did on paper with how they used the material. Yes, this process takes longer than just telling them what they did wrong. But it's also one of the most powerful steps in the passage towards abstraction because it allows them to understand where they're making mistakes and take ownership of the corrections.

START WITH ONE THING

It's normal to feel overwhelmed at the thought of guiding this process from start to finish. The preparation of the adult includes learning how to look at your child, pick one thing to start with, and put everything else on a mental shelf for later. So, where's your child? What's the one concept that you can start with that will help them make progress? Is it the Golden Beads? The Stamp Game? Do they need memorization games or mental computation strategies? Sit with them, work with them, talk with them, observe them. Be a partner in their development, and they'll show you the way.